

Proof of Collatz Conjecture: The Traps, a Flowchart, Their IDs (Number Identification), a Table or Summary, Collatz Conjecture Inequality and Analysis

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Abstract: The author used steps to show that all number using $3n + 1$ conjecture will fall to 4, 2 and eventually 1. And that only one cycle exists. The positive integers may lie to patterns. These could be called traps. A flowchart was used. Each number will have a particular ID dictated by the flowchart. Then an analysis explaining that it ends in 1 and only having one cycle – the 4, 2 and 1 cycle. The number could reach “infinity” but still has a tail.

Keywords: Collatz conjecture, Flowchart, Generalized Collatz Conjecture, Identification, Inequality.

1. INTRODUCTION

There are many analyses to the collatz conjecture.

Statement of the problem.

Consider the following operation on an arbitrary positive integer:

If the number is even, divide it by two.

If the number is odd, triple it and add one.

The Collatz conjecture is: This process will eventually reach the number 1, regardless of which positive integer is chosen initially. [1]

The trivial 4;2;1 cycle

1 is odd so applying $3n + 1$ yields 4 then 4 is even so 4 divided 2 is 2 then 2 is even so 2 divided by 2 is 1. This is a loop by the conjecture. The only known loop.

Steiner (1977) proved that there is no 1-cycle other than the trivial (1;2). [2]

Traps

Traps are used to determine that all numbers will fall to the initial traps. Showing only legal steps for a particular number.

The first traps are

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, ...

Let us call this A.

It is because of the validity of the endless $n/2$ until it reaches 1.

The second traps are

1, 5, 21, 85, 341, ...

$(1-1)/3$

$(2-1)/3$

$(4-1)/3 = 1$ - ok

$(8-1)/3$

$(16-1)/3 = 5$ - ok

$(32-1)/3$

$(64-1)/3 = 21$ - ok

$(128-1)/3$

$(256-1)/3 = 85$ - ok

$(512-1)/3$

$(1024-1)/3 = 341$ - ok

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Let us call this B.

It is because applying $3n+1$ part to the set A.

The third traps are

2, 10, 42, 170, 682, ...

$1 \times 2 = 2$

$5 \times 2 = 10$

$21 \times 2 = 42$

$85 \times 2 = 170$

$341 \times 2 = 682$

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Let us call this C.

It is because applying $n/2$ part to the set B.

The fourth traps are

28, ...

$(1-1)/3$

$(5-1)/3$

$(21-1)/3$

$(85-1)/3 = 28$ - ok

$(341-1)/3$

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Let us call this D.

It is because applying $3n+1$ part to the set B.

The fifth trap or set is applying $n/2$ part to the set C.

4, 20, 84, 340, 1364, ...

$$2 \times 2 = 4$$

$$10 \times 2 = 20$$

$$42 \times 2 = 84$$

$$170 \times 2 = 340$$

$$682 \times 2 = 1364$$

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Let us call this E.

The sixth traps are

3, 27, ...

$$(2-1)/3$$

$$(10-1)/3 = 3 - \text{ok}$$

$$(42-1)/3$$

$$(170-1)/3$$

$$(682-1)/3 = 227 - \text{ok}$$

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Let us call this F.

It is because applying $3n+1$ part to the set C.

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The traps belong to an infinite set.

Traps = AUBUCUD...

The question is in which trap does a number belong. It all started with trap A.

Note: We could use A_1, A_2 , etc. to represent other sets.

Logic

A number could proceed to either $(n/2)$ or $(3n+1)$ part until it reaches 1.

A $(n/2) = A$, all even except 1.

A $(3n+1) = B$, all odd.

B $(n/2) = C$, all even.

B $(3n+1) = D$

C $(n/2) = E$

C $(3n+1) = F$

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Flowchart

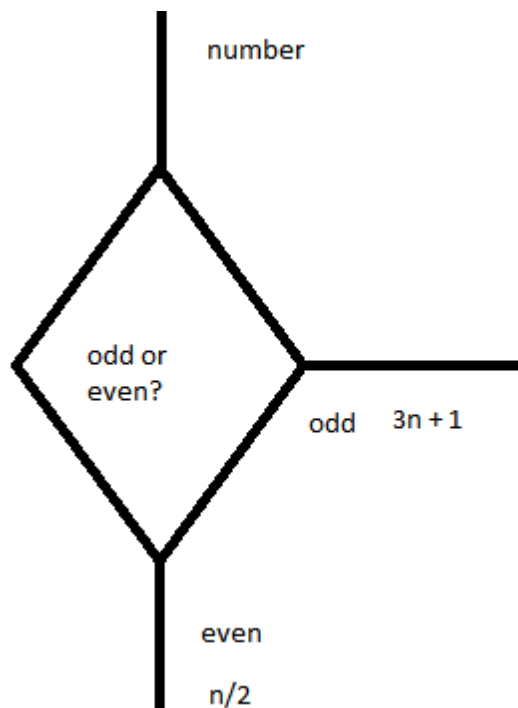


Figure 1 (Flowchart for the mother steps)

1 = odd 1 (this is a proposed id for 1)

How many odd step/s is there to take?

4 is the lowest possible $3n + 1$ result.

2 = even 1 (this is a proposed id for 2)

How many even step/s is there to take?

1 is the lowest possible $n/2$ result.

They are the mother steps.

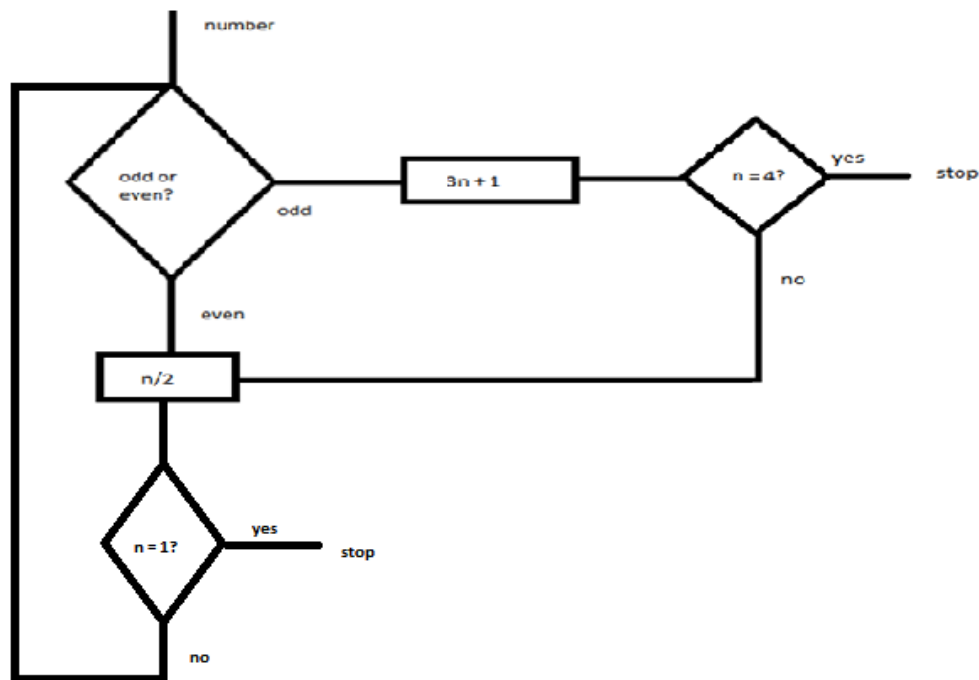


Figure 2 (Flowchart for the conjecture)

Breakdown of numbers or Identification

2 is even 1 or mother step for even numbers and 1 is odd 1 or mother step for odd numbers.

So, here are the breakdown of some numbers:

4 = even 2

Which is an even step then even 1.

8 = even 3

Which is an even step then even 2.

16 = even 4

Which is an even step then even 3.

32 = even 5

Which is an even step then even 4.

64 = even 6

Which is an even step then even 5.

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These are all part of Trap A (1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, ...).

5 is odd so perform odd 1, then becomes 16 or odd 1 even 4.

21 is odd so perform odd 1 then become 64 or odd 1 even 6.

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These are all part of Trap B (1, 5, 21, 85, 341, ...)

10 is even so perform even 1 then becomes 5 or even 1 odd 1 even 4.

42 is even so perform even 1 then becomes 21 or even 1 odd 1 even 6.

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These are all part of Trap C (2, 10, 42, 170, 682, ...).

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Table or Summary

ID	Number/s	Comment
Odd 1	1	mother step
Even 1	2	mother step
Odd 1 Even 1	nonexistent	$2 = (n \times 3) + 1$
Even 1 Even 1 or Even 2	4	simplifying even
Even n	Those divisible by 2^n	A decreasing pattern
Odd 1 Even 2	1	The flowchart could be revised to have no stop at $n=4$.
Even 3	8	
Odd 1 Even 3	non existent	$8 = (n \times 3) + 1$
Odd 1 Even 4	5	$16 = (n \times 3) + 1, n = 5$
Even 1 Odd 1 Even 4	10	$5 \times 2 = 10$
Odd 1 Even 1 Odd 1 Even 4	3	$10 = (n \times 3) + 1, n = 3$
Even 1 Odd 1 Even 1 Odd 1 Even 4	6	$3 \times 2 = 6$
Three scenarios for the starting numbers greater than 5:		
For even numbers,		
Even n		Already enumerated.
Even $n_1 \dots$ Odd 1 Even $n_2 (\dots$ Odd 1 Even 4)		There could be or there are repetition/s of Odd 1 Even n but may not be the same.
For odd numbers,		
Odd 1 Even $n_1 \dots$ Odd 1 Even $n_2 (\dots$ Odd 1 Even 4)		Just an addition of Odd 1.

Notes:

1. The “infinity” of the ID depends on not hitting a trap.
2. () parenthesis in this case means omission. – linguistic possible reinterpretation
3. Some combinations do not exist.

Collatz Conjecture Inequality: $(n - 1)/3 < (n \times 2)$

One Odd step < Two Even steps (except for 1.)

Sample:

	$(3n + 1)$	$(n/2)$	$(n/2)$
1	4	2	1
11	34	17	
21	64	32	16
31	94	57	
41	124	62	31
51	154	77	
61	184	92	46
...			

Note: Division is repeated subtraction. It will certainly reach 1 in one way or another.

Two immediately succeeding even steps negate a preceding odd step. Since an odd step is initially followed by an even step, this is the deciding factor. How much total even steps to negate the odd steps? And odd steps signifies going up. Unless those going up is not negated, this may reach "infinity".

2. ANALYSIS

Observations and

1. Only the tail has a cycle. No number is disjoint. Having another cycle(s) make those numbers disjoint.
2. Division is repeated subtraction. It will certainly reach 1.
3. The (an) IDs or identifications are (could be) infinite. All numbers are connected. Number will have an ending or a tail.

It is either

Odd 1 which is 1,

Or

Even1 which is 2

Even1 Even1 or Even 2 which is 4

Or

...Even 2 (This could be endless.)

Note: This thus support that collatz could go infinite but still has a tail.

4. An Odd step is immediately followed by an Even step thus making the even step more rampant generally. Even step could be infinitely continuous.

5. Collatz conjecture is a no gap conjecture. All numbers are interconnected. We could also check for $5n+1, 7n+1, \dots (n+1$ is a mother conjecture.) We could also check for $n+3, n+5, \dots 3n+3, \dots$

For $n+1: (n - 1) < (n \times 2)$

For $n+5: (n - 1)/5 < (n \times 2)$

...

Note: What generalization it will be with gap or reaching infinity and having cycle(s)?

For the cycle, example: 5, 17, 25, 5 could be cycle, so if you put it in a number line, it will be disconnected to other numbers unless the last number is 25. In here, 4, 2 and 1 is the cycle which is the tail. 4 is connected to number 2 and 8. You can have a cycle in the tail or the head. The head or last number is infinity. So the only option is the tail. All middle cycle makes a disjoint such as for example 5, 17, 25, 5.

For the gap, it is clearly stated that problem asks for any other cycles. Meaning all positive integers are included. A number is connected to two numbers. This is a trivial examination. All positive integers are connected to two numbers.

For reaching infinity, yes indeed the number could reach “infinity” as the ID suggest, the number of IDs could reach infinity but still it has a tail. It reaches 1. The conjecture has a cycle and it should be identified. For it would be just counting up or down. It is also evident in the flowchart that there is a cycle.

3. RESULTS

1. The loop will not stop unless it exits in one of the two situations. Unless the final number be a 4 or a 1.
2. The first step is legality. It should be a positive integer. Then the next step is the recognition if odd or even. Then, a further complex step - formulas to reach 4 or 1. This is the heart for the generalization.
3. The flowchart shows importance of the decision of odd and even. All will fall to that part. Each conjecture may have or should have a flowchart.

4. DISCUSSIONS

1. These 6 proofs may asked to be disproven.
2. Maybe perhaps same analysis or format for the generalized conjecture.

5. CONCLUSION

The traps are correct in getting all the numbers. The flowchart is correct in testing all the numbers. All number will fall to the flowchart. The ID of each number is correct in showing all numbers. The Analysis is correct. And it shows only one cycle. The inequality states that it needs 2 even steps for an odd step. The author also wishes to solve the generalized conjecture as well. This paper is rejected by springer. They are outdated. Things are not meant to be married.

REFERENCES

- [1] O'Connor, J.J.; Robertson, E.F. (2006). "Lothar Collatz". St Andrews University School of Mathematics and Statistics, Scotland.
- [2] Steiner, R. P. (1977). "A theorem on the syracuse problem". Proceedings of the 7th Manitoba Conference on Numerical Mathematics. pp. 553–9.